ON THE USE OF WIGNER DISTRIBUTION IN ULTRASONIC NDE

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INTRODUCTION

The application of signal processing techniques to nondestructive evaluation(NDE) has proven successful for years. The research on this issue has been directed into two types of techniques, namely, the time-domain methods and frequency methods. Techniques of frequency-domain analysis such as Fourier transform and power spectral density reveal important information concerning decomposition of frequency components which may not be available from the time-domain analysis. However, in the case of ultrasonic NDE, the observed echo from a material can be viewed as a superposition of successive time arrivals of different components each characterized by different spectral contents. Both the time domain waveform and the spectrum can be very confusing and are simultaneously relevant for a complete description of a signal [2]. Neither the spectral analysis nor the time-domain methods alone can provide needed information. Therefore, mixed tools are often necessary to be introduced to overcome the limits imposed on these two mutually complementary families of techniques. These tools are generally referred to as time-frequency distribution or time-frequency analysis and have been found useful in numerous applications such as speech processing , sonar analysis , detection of electrocardiography (EEG) signals and X-ray diffraction. Among them, the Wigner distribution has been studied intensively in the last few years due to its high resolution property.

The paper begins with a mathematical description of the Wigner distribution and then examines its new application to the problems of ultrasonic nondestructive evaluation. An example with artificial data is given to demonstrate its capability. The experimental results are very complex and confusing at the first glance. However, after carefully investigating the mathematical mechanism of the Wigner distribution, interpretation is made possible and has shown encouraging results.

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The definition of the time-varying correlation of a nonstationary complex stochastic process s(t) is [3]

$$R(t,\tau)= \overline{s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})} \tag{1}$$

where the overbar denotes an ensemble average. The "center" time in (1) is t while the "separating" time is τ. However, if an ensemble is not available, or if s(t) is a deterministic waveform, the extension of equation (1) is simply

$$R(t,\tau)= s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2}) \tag{2}$$

The associated "spectrum" at time t can be obtained by Fourier transforming (2) on τ

$$\begin{aligned} W(t,f) &= \int_{-\infty}^{\infty} R(t,\tau)\exp(-j2\pi f\tau)d\tau \\ &= \int_{-\infty}^{\infty} s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})\exp(-j2\pi f\tau)d\tau \end{aligned} \tag{3}$$

This time-frequency function W(t,f) is called the Wigner distribution (WD).

<u>Property 1</u> : It is a real function, even when s(t) is complex.

<u>Property 2</u> : Product and Convolution

Let s(t) = a(t) b(t), then, the WD of s(t) is (inserting subscripts as needed)

$$W_s(t,f) = \int_{-\infty}^{\infty} W_a(t,v)W_b(t,f-v)dv = W_a(t,f)\overset{f}{\otimes}W_b(t,f)$$

which is a convolution on frequency f, for fixed t. In a similar fashion, if s(t) is the convolution in time of two other signals,

$$s(t) = a(t) \overset{t}{\otimes} b(t) = \int_{-\infty}^{\infty} a(\tau)b(t-\tau)d\tau , \tag{4}$$

then the WD of s(t) is

$$W_s(t,f) = W_a(t,f) \overset{t}{\otimes} W_b(t,f) = \int_{-\infty}^{\infty} W_a(\tau,f)W_b(t-\tau,f)d\tau$$

which is a convolution on time t, for fixed f.

<u>Property 3</u>:

Integration on (3) yields the marginals

$$\int_{-\infty}^{\infty} W(t,f)dt= |S(f)|^2 \tag{5}$$

$$\int_{-\infty}^{\infty} W(t,f)df = |s(t)|^2 \tag{6}$$

where S(f) is the Fourier transform of s(t). That is, the summing up over all times at a particular frequency gives the energy density spectrum, and the summing up of the energy distribution for all frequencies at a particular time gives the instantaneous energy [4].

The total energy E, expressed in terms of the distribution, is given by

$$E = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(t,f)dtdf = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df \tag{7}$$

Property 4 : Alternative form of WD

In an alternative form, W(t,f) can be expressed in terms of S(f)

$$W(t,f) = \int_{-\infty}^{\infty} S(f+\frac{v}{2})S^*(f-\frac{v}{2})\exp(j2\pi vt)dv \tag{8}$$

Since the integrand in the above equation always introduces unnecessary correlation between the positive sideband and its negative image of a real signal and results in cross-term interference in the baseband of the WD, the original signal is sometimes transfomed to its analytic signal before being calculated.

AN EXAMPLE WITH ARTIFICIAL DATA

In this section we examine the WD of a single tone FM wave given by

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \tag{9}$$

It can be shown [1] that the WD of s(t) is

$$\begin{aligned} W(t,f) = & \sum_{k=-\infty}^{\infty} \left\{ C_k \delta(f-f_c-kf_m) + D_k \delta[f-f_c-(k+\frac{1}{2})f_m] \right\} \\ & + \sum_{k=-\infty}^{\infty} \left\{ C_k \delta(f+f_c+kf_m) + D_k \delta[f+f_c+(k+\frac{1}{2})f_m] \right\} \\ & + \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \mathrm{Re}\{ \exp(j2\pi t(2f_c+kf_m+nf_m)) J_n(\beta) J_k^*(\beta) \} \; \delta(2f-nf_m+kf_m) \end{aligned} \tag{10}$$

where

$$C_k = J_k(\beta)J_k^*(\beta) + 2 \sum_{n=k+1}^{\infty} \mathrm{Re}\{ \exp(j4\pi f_m(k-n)t) J_n(\beta) J_{2k-n}^*(\beta) \} \tag{11}$$

$$D_k = 2 \sum_{n=k+1}^{\infty} \mathrm{Re}\left\{ \exp(j4\pi f_m(k-n+\frac{1}{2})t) J_{2k-n+1}(\beta) J_n^*(\beta) \right\} \tag{12}$$

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(\, j\, (\beta \sin x - nx))dx \tag{13}$$

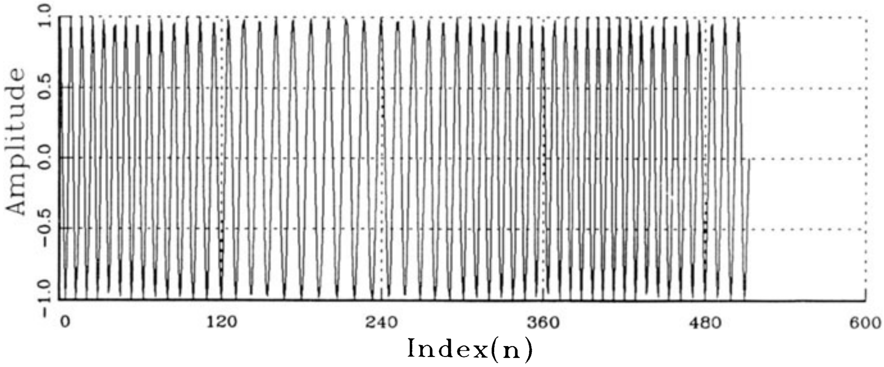

Figure 1. Single-tone FM wave.

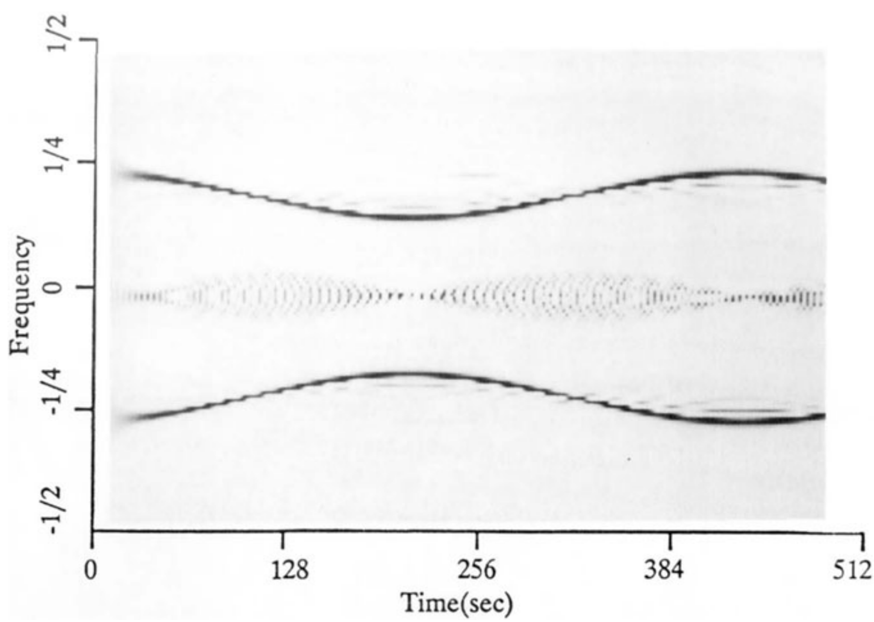

Figure 2. WD of a single-tone FM wave.($A_c = 1$, $f_c = 0.1$, $f_m = 0.0025$, $\beta = 10$ for $0 \leq t \leq 512$)

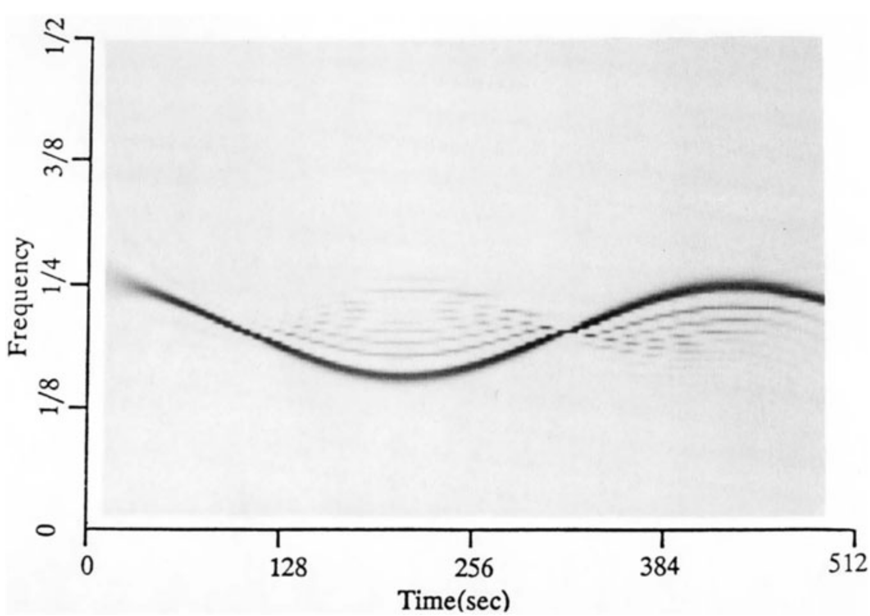

Figure 3. WD of the analytic signal of a single-tone FM wave. (Same parameter as above)

The third term in equation (10) is the cross-component which appears in the baseband of the WD in figure 2 due to the effect of the correlation multiplication in the kernel function. The first two terms are the major contributions to the distribution. After transforming s(t) into its analytic signal, the WD becomes single-side-banded and the cross-component in the baseband is also removed.(see figure 3). As has been expected, the WD appears to be a frequency demodulator as one may see the sinusoidal wave in figure 2 and 3.

EXAMPLES WITH ULTRASONIC NDE DATA

The ultrasonic signals used for the simulation are part of a larger simulated bond A-scan test data set obtained from the Army's Material Technology Laboratory (Watertown, MA) . E1 and E5 are pulse echoes from a simulated bondline reflector with transducer center frequencies being 5 and 30 MHz respectively. The thickness of the reflector is 0.0192". D1 and D5 were obtained by the same experiment setup with transducer frequencies being 5 and 30 MHz respectively except that the thickness of the reflector is 0.0136". The experimental results are shown in figure 5 and figure 6. Since the WD is not always positive, only the positive part is displayed.

As one may observe, the results consist of equally spaced vertical columns. Each column except the first one and the last one is consisted of equally spaced short horizontal strips. The odd number columns are auto-components corresponding to the equally spaced pulses in the time signal. The even number columns are cross-components centered in the middle of each adjacent pulses. It has been shown [1] that the WD for a fixed time t can be approximated by

$$W(t,f) = K + 2\,|C(t,f)|\cos[4\pi fT-\phi(t,f)] \tag{14}$$

when t lies in the duration of the pulses and

$$W(t,f) = 2\,|C(t,f)|\cos[2\pi fT-\phi(t,f)], \tag{15}$$

when t lies in the duration of a cross-component. Here $C(t,f)$ is the Fourier transform of a short ultrasonic pulse and $\phi(t,f)$ is its associated phase. K is a constant and T is the time interval between two adjacent pulses.

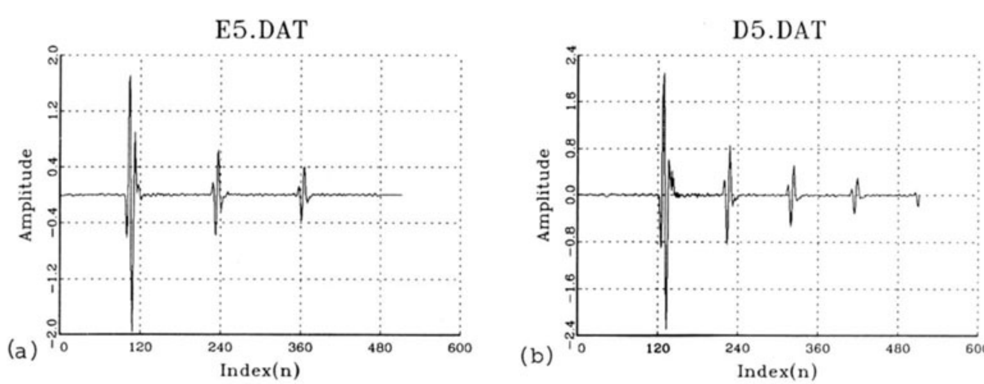

Figure 4. (a) E5 (b) D5.

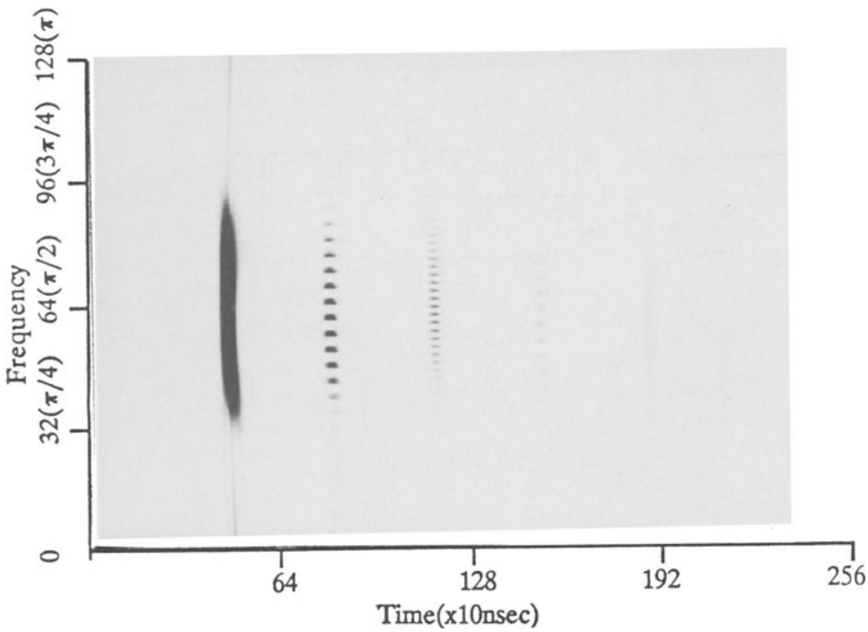

Figure 5. WD of E5

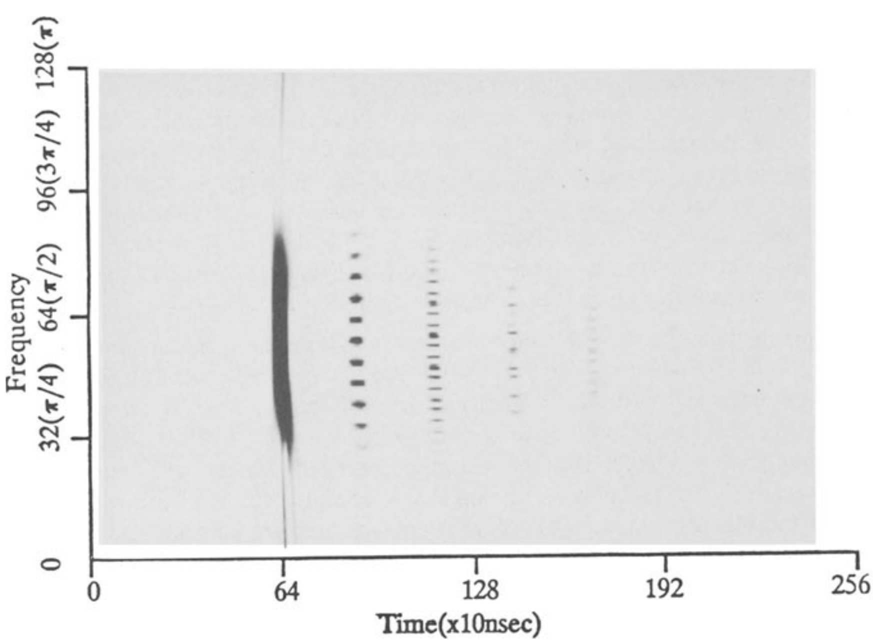

Figure 6. WD of D5.

Table 1

	Transducer Frequency	Defect Thickness	Number of Zero-Crossings	Measured time interval between pulses
E5	30 MHz	0.0192"	69	690ns
D5	30MHz	0.0136"	47	470ns

By the above argument, the bondline thickness can be easily found by using equation (14) or equation (15) to calculate the time interval T elapsed between each two adjacent pulses. Since it is easier to find the number of zero crossings in a time series than to find the number of local minimums, equation (15) is used to obtain the above table.

The above example clearly illustrates one of many potential applications of WD in ultrasonic NDE.

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